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Using the Gould–DeWitt scheme to approximate the dynamic collision frequency in a dense electron gas

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Abstract

In order to describe dielectric properties in dense plasmas, a consistent calculation of the collision frequency is required. We present new calculations for an electron gas at parameters which are relevant for warm dense matter. In particular, we focus on the influence of the different approximations for the collision frequency in the Gould–DeWitt scheme. We use the dynamic collision frequency in the Born, Lenard–Balescu and ladder approximation. The inclusion of collisions in a consistent manner modifies, e.g., the dielectric function significantly in the warm dense matter regime.

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1. Introduction

For a charged particle system, the longitudinal dielectric function $\epsilon(\vec{k}, \omega)$ contains important information about different physical properties. It can be directly related to the wavevector ($\vec{k}$) and frequency (ω) dependent conductivity $\sigma(\vec{k}, \omega)$ describing transport phenomena via $\epsilon(\vec{k}, \omega) = 1 + \mathrm{i}\sigma(\vec{k}, \omega)/(\epsilon_0\omega)$. In particular, optical properties such as refraction index, absorption coefficient, reflectivity and bremsstrahlung are obtained considering the long wavelength limit of the dielectric function $\epsilon(0, \omega)$ or the dynamical conductivity $\sigma(0, \omega) = \sigma(\omega)$, respectively, (see [1–3]).

The simplest approximation for the dielectric function is the random phase approximation (RPA) for a collisionless plasma. Going beyond the RPA in the long-wavelength limit, the dielectric function is related to the complex valued dynamic collision frequency via a generalized Drude expression [2, 4]

$$\epsilon(\omega) = 1 - \frac{\omega_{\mathrm{pl}}^2}{\omega[\omega + \mathrm{i}\nu(\omega)]}, \qquad (1)$$

with the electronic plasma frequency $\omega_{\mathrm{pl}}^2 = n_{\mathrm{e}}e^2/(\epsilon_0 m_{\mathrm{e}})$. In [5, 6], we have demonstrated that it is necessary to go beyond the RPA by including collisions in order to obtain reliable results for plasma properties in the warm dense matter region. The dynamic collision frequency $\nu(\omega)$ has to be calculated taking into account the effects of dynamic screening and strong collisions. In this paper, we apply the Gould–DeWitt [7] scheme to determine the dynamic collision frequency for a free electron plasma.

2. Dynamic collision frequency

Within the Zubarev approach of a linear response theory, the dynamic collision frequency is expressed according to

$$\nu(\omega) = \frac{\Omega_0}{\epsilon_0 k_B T \omega_{\rm pl}^2} \langle \mathbf{J}_k^{el}; \mathbf{J}_k^{el} \rangle_{\omega + {\rm i}\eta}, \qquad (2)$$

where $\mathbf{J}_k$ is the electric current operator and Ω_0 is a normalization volume. The angle brackets denote the equilibrium correlation function (see [2]). This expression was evaluated using a consistent many-particle theory. In Born approximation we obtain [2]

$$\nu^{\rm Born} = -{\rm i} \frac{\epsilon_0 n_i \Omega_0^2}{6\pi^2 e^2 n_e m_e} \int_0^\infty {\rm d}q\, q^6 V(q)^2 S_{ii}(q) \frac{1}{\omega} [\epsilon_{\rm RPA}(q,\omega) - \epsilon_{\rm RPA}(q,0)], \qquad (3)$$

where $V(q)$ is the pure Coulomb potential, $\epsilon_{\rm RPA}$ is the RPA dielectric function. Since we consider a singly charged plasma, the electron and ion density n_i and n_e, respectively, are identical. The static ion structure factor $S_{ii}(q)$ accounts for ion correlations, which are particularly important in highly ionized materials. Equation (3) gives the electron–ion collision frequency. In order to include electron–electron collisions, higher orders in the expansion of the correlation functions have to be considered [2, 3].

Dynamic screening can be treated by a partial summation of loop diagrams introducing a screened interaction when considering the polarization function. In adiabatic approximation, the collision frequency reads [2]

$$\nu^{\rm LB}(\omega) = {\rm i} \frac{\epsilon_0 n_i \Omega_0^2}{6\pi^2 e^2 n_e m_e} \int_0^\infty {\rm d}q\, q^6 V(q)^2 S_{ii}(q) \frac{1}{\omega} \left[\epsilon_{\rm RPA}^{-1}(q,\omega) - \epsilon_{\rm RPA}^{-1}(q,0)\right]. \qquad (4)$$

This treatment corresponds to solving the linearized Lenard–Balescu equation, which is denoted by the index LB. The ion structure factor in equations (3) and (4) is assumed to be $S_{ii}(q) = 1$. For the inclusion of strong collisions, a systematic treatment is obtained by performing a summation of ladder diagrams with respect to a statically screened Coulomb potential. The real part of the collision frequency in ladder approximation can be expressed by an integral over the momentum space and summation over the angular momentum [2]:

$$\begin{aligned} {\rm Re}\, \nu^{\rm Ladder}(\omega) = \frac{\beta \Omega_0^2}{3\pi^3 \hbar} \int_0^\infty {\rm d}p p^2 \int_0^\infty {\rm d}p' p'^2 \frac{1 - {\rm e}^{-\beta\hbar\omega}}{\beta\hbar\omega} \delta\left(p^2 - p'^2 + \frac{2m_e\omega}{\hbar}\right) f_p^e \sum_{l=0}^\infty (l+1) \\ \times \left[\left\{p' T_l^-(p,p';E_p^e) - p T_{l+1}^-(p,p';E_{p'}^e)\right\}\right. \\ \times \left\{p' T_l^+(p',p;E_p^e) - p T_{l+1}^+(p',p;E_{p'}^e\right\} \\ + \left\{p T_l^-(p,p';E_{p'}^e) - p' T_{l+1}^-(p,p';E_p^e)\right\} \\ \left.\times \left\{p T_l^+(p',p;E_{p'}^e) - p' T_{l+1}^+(p',p;E_p^e)\right\}\right], \end{aligned} \qquad (5)$$

where f_p^e is the Maxwell–Boltzmann distribution function and $T_l(p,p';E_p)$ is the half-off-shell T-matrix with respect to the angular momentum l. The imaginary part of the collision frequency was calculated via the Kramers–Kronig relation.

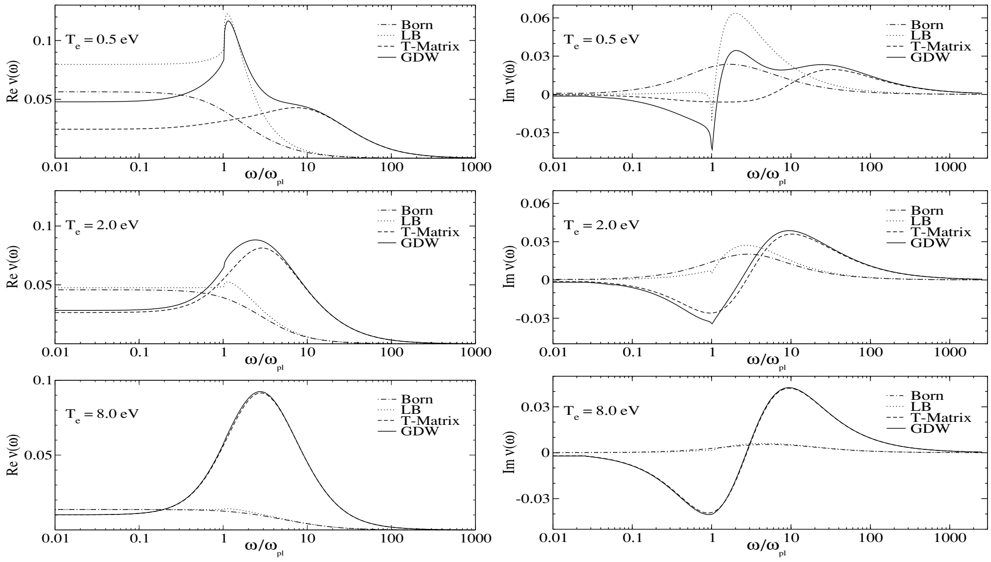

Figure 1. Re ν (left) and Imν (right) for plasmas with a density of 10^{21} cm^{-3} and temperatures of 0.5 eV, 2.0 eV and 8.0 eV.

3. The Gould–DeWitt approach

To account for screening as well as strong collisions, both types of correlations should be combined within a dynamically screened T-matrix approximation. However, because of the multifrequency character of such an approximation, no tractable expressions for the correlation functions have been derived so far. Following Gould and DeWitt [7], both effects are combined by adding up the two contributions. As an approximation for the dynamically screened T-matrix expression, we decompose $\nu(\omega)$ according to [2]

$$\nu^{\mathrm{GD}}(\omega) = \nu^{\mathrm{ladder}}(\omega) + \nu^{\mathrm{LB}}(\omega) - \nu^{\mathrm{Born}}(\omega). \quad (6)$$

The Born approximation is taken into account dynamically (LB). Since the T-matrix with respect to a statically screened potential also contains a contribution in the Born approximation, the Born approximation with respect to the statically screened Coulomb potential is subtracted in order to avoid double counting. This so-called Gould–DeWitt approach has been widely used to treat the effects of distant as well as of close collisions in a dense plasma (see [8–10]).

We calculate the dynamic collision frequency, equation (6), for a density of $n_e = 10^{21}$ cm^{-3} and three different temperatures of $T_e = 0.5$ eV, 2.0 eV and 8.0 eV. The influence of collisions is most important in this domain as shown in [5, 6]. In figure 1, the real parts of the dynamic collision frequency are displayed on the left side. Besides the Gould–DeWitt result, the contributions from dynamic screening, equation (4), strong collisions, equation (5), as well as the statically screened Born approximation, equation (3), are compared. In the high-frequency limit, the dynamic and static screening results almost coincide, since Re $\epsilon(q, \omega) \approx 1$. As a consequence, the Gould–DeWitt result is dominated by the contribution of strong collisions. For lower temperatures, the influence of the dynamically screened potential is very strong. The peak close to the plasma frequency is due to plasmon excitations. The influence of the Lenard–Balescu term near the plasma frequency decreases with higher temperatures. For the highest temperature of 8 eV, strong collisions dominate the Gould–DeWitt result. The imaginary parts of the collision frequency are shown on the right side of figure 1. As for the real part, the influence of dynamic screening is most important at low temperatures for frequencies close to the plasma frequency. For higher frequencies, the Gould–DeWitt result is again dominated by the contribution of strong collisions. For the considered density and temperature domain, we conclude that the modifications of the collision frequency within the Gould–DeWitt scheme are important for a correct description of the correlations.

4. Conclusions

We have shown that the influence of dynamic screening and strong collisions in the warm dense matter region is significant. For low temperatures, the contribution of the Lenard–Balescu collision frequency is important and for higher temperatures strong collisions dominate. In the next step, the influence of higher moments of the distribution function on the collision frequency will be treated by a renormalization factor [2]. These results can be used to further investigate the Thomson scattering in dense plasmas [5, 6, 11]. The dynamic structure factor relevant for the description of Thomson scattering and directly related to the dielectric function is strongly influenced by collisions in a region with a degeneracy parameter $\Theta \approx 1$ and coupling parameters $\Gamma \geqslant 1$. In particular, this applies to conditions relevant for the next stage of the free electron laser at DESY Hamburg. Therefore, the use of Thomson scattering as a diagnostic tool for warm and dense matter requires to go beyond the standard RPA description and to account for a dynamic collision frequency. For a complete description of Thomson scattering, ions and bound states have to be considered as well.

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